CHAPTER 13

# Investment

Investment is one of the most important macroeconomic aggregates, for both short-run and long-run reasons. In the short run, it turns out to be much more volatile than other components of aggregate demand, so it plays a disproportionate role in business cycles. In the long run, investment is capital accumulation, which we have seen is one of the main determinants of output and output growth. That is why we now turn to an inquiry into the determinants of investment.

We have often considered the problem of a firm in partial equilibrium, and analysed how it chooses its optimal level of capital. The firm owns or rents capital, and it can borrow and lend at a fixed interest rate. We saw over and over again that the optimal level of the capital stock for such a firm implies that the marginal product of capital (MPK) equals that interest rate. Investment will thus be whatever is needed to adjust the capital stock to that desired level.

But is that all there is to investment? There are many other issues: the fact that investments are often irreversible or very costly to reverse (e.g. once you decide to build a new plant, it is costly to get rid of it), and that there are costs of installing and operating new equipment. Because of such things, a firm will have a much harder problem than simply immediately increasing its capital stock in response to a fall in the interest rate.

We will now deal with some of these issues, and see how they affect some of the conclusions we had reached in different contexts. We will start by looking at standard practice in corporations and assess the role of uncertainty. We will then put aside the role of uncertainty to develop the Tobin's q theory of investment.

## 13.1 | Net present value and the WACC

The weighted average cost of capital (WACC) is defined as a weighted average of the firm's cost of financing through equity and debt. If a project yields a return higher than the WACC, it is more likely to be implemented.

The best place to start our understanding of investment is to go where all corporate finance books start: investment is decided on the basis of the net present value (NPV) of a project. If a project is started in period 0 and generates a (positive or negative) cash flow of $W_t$ in any period $t$ up until time $T$, the NPV will be given by:

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$$NPV = \sum_{t=0}^{T} \frac{1}{(1+r)^t} W_t, \tag{13.1}$$

where $r$ is the cost of capital. Typically, one would expect to have a negative cash flow initially, as investment is undertaken, before it eventually turns positive.

The key question is whether this NPV is positive or not. If NPV $> 0$, then you should invest; if NPV $< 0$, then you shouldn't. This sounds simple enough, but this immediately begs the question of what interest rate should be used, particularly considering that firms use a complex mix of financing alternatives, including both equity and debt to finance their capital expenditure (CAPEX) programs. In short, what is the cost of capital for a firm? A very popular measure of the cost of capital is the so-called weighted average cost of capital (WACC), which is defined as

$$WACC = \alpha_{eq} r_{eq} + \left(1 - \alpha_{eq}\right) r_{debt}, \tag{13.2}$$

$$\alpha_{eq} = \left(\frac{\text{Net worth}}{\text{Total Assets}}\right). \tag{13.3}$$

Here, $r_{eq}$ is the return on equity (think dividends) and $r_{debt}$ is the return on debt issued (think interest). This is a very popular model in part because it is easy to compute. Basically, it allocates the cost of equity using as weight the fraction of your assets that the firm finances with equity. The return on equity can, in turn, be easily derived, say from a CAPM regression, as explained in the previous chapter. With a weight equal to the share of assets that you finance with debt, the formula uses the cost of debt. For this, you may just take the interest cost of the company's issued debt.

In practice, typically firms go through a planning cycle in which the CFO sets a WACC for the following planning cycle and units decide on those projects that have a return higher than that WACC. Only those that have a return higher than the cost of capital get the green light to go ahead. There are several issues with this procedure. Units tend to exaggerate the benefits of their projects to obtain additional resources, projects take a lot of time, and there are many tax-induced distortions (such as reporting investments as expenses to get a tax credit). A lot of corporate finance is devoted to exploring these and other issues.

### 13.1.1 | Pindyck's option value critique

Investment is irreversible, so there is a significant option value when investing. This section illustrates how the value to wait can be essential in evaluating the attractiveness of investment projects.

Investment is a decision in which the presence of uncertainty makes a critical difference. This is because investment is mostly irreversible. It follows that there are option-like features to the investment decision that are extremely relevant. Consider, for example, a project with an NPV of zero. Would you pay more than zero for it? Most probably yes, if the return of the project is stochastic and you have the possibility of activating the project in good scenarios. In other words, a zero NPV project has positive value if it gives you the option to call it when, and only when, it makes you money. In that sense, it is just like a call option – i.e. one in which you purchase the right to buy an asset at some later date at a given price. People are willing to pay for a call option. This line of reasoning is critical, for example, in the analysis of mining rights and oil fields. Today an oil field may not be profitable, but it sure is worth more than zero if it can be tapped when energy prices go up.[1]

One of the most studied implications of this option-like feature is when there is an option value to waiting. This is best described by an example such as the following. Suppose you can make an initial investment of \$2200, and that gives you the following stochastic payoff:

$$
\begin{array}{cccc}
t=0 & & t=1 & t=2 \\
 & q & P_1 = 300 & \dots \\
P_0 = 200 & & & \\
 & 1-q & P_1 = 100 & \dots
\end{array}
$$

That is, in the first period you make \$200 for sure, but from then onward the payoff will be affected by a shock. If the good realisation of the shock occurs (and let's assume this happens with probability $q$), you get \$300 forever. If you are unlucky, you get \$100 forever instead. Suppose $q = 0.5$ and $r = 0.10$. Given this information, the expected NPV of the project, if it is considered at $t = 0$, is

$$NPV = -2200 + \sum_{t=0}^{\infty} \frac{200}{(1.1)^t} = -2200 + 2200 = \$0. \tag{13.4}$$

In other words, this is a really marginal investment opportunity. But now consider the option of waiting one period, so that the uncertainty gets resolved, and the project only happens if the good state gets realised (which happens with a 50% probability). Then we have

$$NPV = 0.5\left[-\frac{2200}{1.1} + \sum_{t=1}^{\infty} \frac{300}{(1.1)^t}\right] = 0.5\left[-\frac{2200}{1.1} + \frac{300}{(1.1)}\left(\frac{1}{1-\frac{1}{1.1}}\right)\right] = 500! \tag{13.5}$$

As can readily be seen this is a much better strategy. What was missing in (13.4) was the option value (of \$500) that the entrepreneur was foregoing by implementing the project in period 0. In other words, she should be willing to pay a substantial amount for the option of waiting until period 1.

This option value argument explains why a firm does not shut down in the first month it loses money – staying open has the value of keeping alive the potential for rewards when things get better. It also explains why people don't divorce whenever marriage hits its first crisis, nor do they quit their job when they have a bad month. These are all decisions that are irreversible (or at least very costly to reverse), and that have uncertain payoffs over time. In sum, they are very much like investment decisions!

A critical lesson from considering the option value entailed by irreversible (or costly reversible) decisions is that uncertainty increases the option value. If there is a lot of variance, then even though the business may be terrible right now, it makes sense to wait instead of shutting down because there is a chance that things could be very good tomorrow. This means that lots of uncertainty will tend to depress the incentive to make investments that are costly to reverse. A firm will want to wait and see instead of committing to building an extra plant; a farmer will want to wait and see before buying that extra plough, and so on. This idea underlies those claims that uncertainty is bad for investment.[2]

## 13.2 | The adjustment cost model

Why don't firms move directly to their optimal capital stock? Time to build or adjustment costs prevent such big jumps in capital and produce a smoother investment profile more akin to that observed in the data. This section introduces adjustment costs to capital accumulation delivering a smoother demand for investment.

Let us now go back to our basic framework of intertemporal optimisation, but introducing the crucial assumption that the firm faces adjustment costs of installing new capital. For any investment that the firm wants to make, it needs to pay a cost that varies with the size of that investment. That is to say, the firm cannot purchase whatever number of new machines it may desire and immediately start producing with them; there are costs to installing the new machines, learning how to operate them, etc. The more machines it buys, the greater the costs.[3]

If that is the case, how much capital should the firm install, and when? In other words, what is optimal investment?

### 13.2.1 | Firm's problem

The firm's objective function is to maximise the discounted value of profits:

$$\int_0^\infty \pi_t e^{-rt} dt, \tag{13.6}$$

where $\pi_t$ denotes firm profits and $r > 0$ is (constant and exogenously-given) real interest rate. The profit function is[4]

$$\pi_t = y_t - \psi\left(i_t, k_t\right) - i_t, \tag{13.7}$$

where $y_t$ is output and $\psi\left(i_t, k_t\right)$ is the cost of investing at the rate $i_t$, when the stock of capital is $k_t$. The term $\psi\left(i_t, k_t\right)$ is the key to this model; it implies adjustment costs, or costs of investing. Notice that if there are no costs of adjustment, given our assumption of a constant and exogenous interest rate, firms should go right away to their optimal stock. This would give an instantaneous investment function that is undefined, the investment rate either being zero or plus or minus infinite. However, in reality it seems that investment decisions are smoother and this has to mean that there are costs that make it very difficult if not impossible to execute large and instantaneous jumps in the stock of capital. Why would there be costs? We can think of several reasons. One is time-to-build; it simply takes time to build a facility, a dam, a power plant, a deposit, etc. This naturally smooths investment over time. Now, if you really want to hurry up, you can add double shifts, more teams, squeeze deadlines, etc., but all this increases the cost of investment expansions. We thus introduce the costs of adjustment equation as a metaphor for all these frictions in the investment process.

What is the equation of motion that constrains our firm? It is simply that the growth of capital must be equal to the rate of investment:

$$\dot{k}_t = i_t. \tag{13.8}$$

The production function is our familiar

$$y_t = Af\left(k_t\right), \tag{13.9}$$

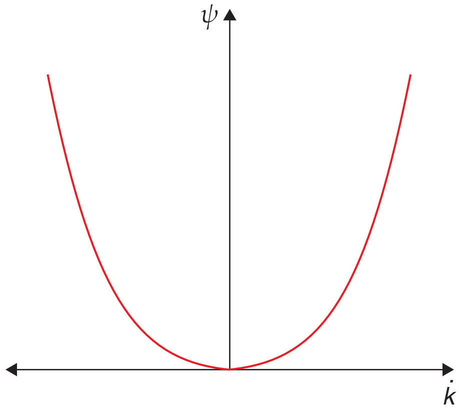
where $A$ is a productivity coefficient and where $f'(\cdot) > 0, f''(\cdot) < 0$, and Inada conditions hold.

Next, specialise the investment cost function to be

$$\psi\left(i_t, k_t\right) = \frac{1}{2\chi}\frac{\left(i_t\right)^2}{k_t}, \tag{13.10}$$

where $\chi > 0$ is a coefficient. Figure 13.1 depicts this function.

**Figure 13.1 Adjustment costs**




The important assumption is that the costs of adjustment are convex, with a value of zero at $i_t = 0$. The latter means that there is no fixed cost of adjustment, which is a simplifying assumption, while the former captures the idea that the marginal cost of adjustment rises with the size of that adjustment.

## Solving this problem

The Hamiltonian can be written as

$$H = Af\left(k_t\right) - \frac{1}{2\chi}\frac{\left(i_t\right)^2}{k_t} - i_t + q_t i_t, \tag{13.11}$$

where $q_t$ is the costate corresponding to the state $k_t$, and the control variable is $i_t$. The first order condition with respect to the control variable is

$$\frac{\partial H}{\partial i_t} = 0 \Rightarrow \frac{1}{\chi}\frac{i_t}{k_t} = q_t - 1. \tag{13.12}$$

The law of motion for the costate is

$$\dot{q}_t = q_t r - \frac{\partial H}{\partial k_t} = q_t r - Af'\left(k_t\right) - \frac{1}{2\chi}\left(\frac{i_t}{k_t}\right)^2. \tag{13.13}$$

The transversality condition is

$$\lim_{T\to\infty}\left(q_T k_T e^{-rT}\right) = 0. \tag{13.14}$$

### 13.2.2 | Tobin's q

Our model of investment delivers the result that investment is positive if the value of capital is larger than the replacement cost of capital. This is dubbed Tobin's q theory of investment in honour of James Tobin, who initially proposed it.

Recall that the costate $q_t$ can be interpreted as the marginal value (shadow price) of the state $k_t$. In other words, it is the value of adding an extra unit of capital. What does this price depend on? Solving (13.13) forward yields

$$q_T = q_t e^{r(T-t)} - \int_t^T \left[ Af'\left(k_v\right) + \frac{1}{2\chi}\left(\frac{i_v}{k_v}\right)^2 \right] e^{r(T-v)} dv. \tag{13.15}$$

Dividing this by $e^{rT}$ and multiplying by $k_T$ yields

$$k_T q_T e^{-rT} = k_T \left\{ q_t e^{-rt} - \int_t^T \left[ Af'\left(k_v\right) + \frac{1}{2\chi}\left(\frac{i_v}{k_v}\right)^2 \right] e^{-rv} dv \right\}. \tag{13.16}$$

Next, applying the TVC condition (13.14), we have

$$\lim_{T\to\infty} \left\{ k_T \left( q_t e^{-rt} - \int_t^T \left[ Af'\left(k_v\right) + \frac{1}{2\chi}\left(\frac{i_v}{k_v}\right)^2 \right] e^{-rv} dv \right) \right\} = 0. \tag{13.17}$$

If $\lim_{T\to\infty} k_T \neq 0$ (it will not be – see below), this last equation implies

$$q_t e^{-rt} - \int_t^\infty \left[ Af'\left(k_v\right) + \frac{1}{2\chi}\left(\frac{i_v}{k_v}\right)^2 \right] e^{-rv} dv = 0, \tag{13.18}$$

or

$$q_t = \int_t^\infty \left[ Af'\left(k_v\right) + \frac{1}{2\chi}\left(\frac{i_v}{k_v}\right)^2 \right] e^{-r(v-t)} dv. \tag{13.19}$$

Hence, the price of capital is equal to the present discounted value of the marginal benefits of capital, where these have two components: the usual marginal product ($Af'\left(k_t\right)$) and the marginal reductions in investment costs that come from having a higher capital stock in the future.

The fact that $q$ is the shadow value of an addition in the capital stock yields a very intuitive interpretation for the nature of the firm's investment problem. A unit increase in the firm's capital stock increases the present value of the firm's profits by $q$, and this raises the value of the firm by $q$. Thus, $q$ is the market value of a unit of capital.

Since we have assumed that the purchase price of a unit of capital is fixed at 1, $q$ is also the ratio of the market value of a unit of capital to its replacement cost. Tobin (1969) was the first to define this variable, nowadays known as Tobin's q.

Notice that (13.12) says that a firm increases its capital stock if the market value of the capital stock exceeds the cost of acquiring it, and vice versa. This is known as Tobin's $q$-theory of investment.

### 13.2.3 | The dynamics of investment

Rearranging (13.12) and using (13.8), we can obtain

$$\dot{k}_t = \chi \left(q_t - 1\right) k_t. \tag{13.20}$$

Next, using (13.20) in (13.13) we have

$$\dot{q}_t = rq_t - Af'\left(k_t\right) - \frac{1}{2}\chi\left(q_t - 1\right)^2. \tag{13.21}$$

Notice that (13.20) and (13.21) are a system in two differential equations in two unknowns, which can be solved independently of the other equations in the system. What is the interpretation of this system? Taking only a minor liberty, we can refer to this as a system in the capital stock and the price of capital.

#### Initial steady state

Return to the system given by equations (13.20) and (13.21). It is easy to show that, in a steady state (i.e. a situation where growth rates are constant), it must be the case that these growth rates must be zero.[5] Let * denote steady state variables. From (13.20), $\dot{k}_t = 0$ implies $q^* = 1$. In turn, setting $\dot{q}_t = 0$ in (13.21) implies $r = Af'(k^*)$, which shows that the marginal product of capital is set equal to the rate of interest. (Have we seen that before?) It follows that, in steady state, firm output is given by $y^* = Af(k^*)$. Note that this is true *only in steady state*, unlike what we had in previous models, where this was the optimality condition everywhere!

#### Dynamics

The system (13.20) and (13.21) can be written in matrix form as

$$\begin{bmatrix} \dot{k}_t \\ \dot{q}_t \end{bmatrix} = \Omega \begin{bmatrix} k_t - k^* \\ q_t - q^* \end{bmatrix}, \tag{13.22}$$

where

$$\Omega = \begin{bmatrix} \left.\frac{\partial \dot{k}}{\partial k}\right|_{SS} & \left.\frac{\partial \dot{k}}{\partial q}\right|_{SS} \\ \left.\frac{\partial \dot{q}}{\partial k}\right|_{SS} & \left.\frac{\partial \dot{q}}{\partial q}\right|_{SS} \end{bmatrix}. \tag{13.23}$$

Note that

$$\left.\frac{\partial \dot{k}}{\partial k}\right|_{SS} = 0 \tag{13.24}$$

$$\left.\frac{\partial \dot{k}}{\partial q}\right|_{SS} = \chi k^* \tag{13.25}$$

$$\left.\frac{\partial \dot{q}}{\partial k}\right|_{SS} = -Af''(k^*) \tag{13.26}$$

$$\left.\frac{\partial \dot{q}}{\partial q}\right|_{SS} = r, \tag{13.27}$$

so that

$$\Omega = \begin{bmatrix} 0 & \chi k^* \\ -Af''(k^*) & r \end{bmatrix}. \tag{13.28}$$

This means that

$$\text{Det }(\Omega) = Af''(k^*)\,\chi k^* < 0. \tag{13.29}$$

We know that Det($\Omega$) is the product of the roots of the matrix $\Omega$. If this product is negative, the roots must have different signs. Note that $q_t$ is a jumpy variable, and $k_t$ is a sticky variable. With one positive root and one negative root, the system is saddle-path stable. Figure 13.2 shows the corresponding phase diagram.

Suppose starting at the steady state, the productivity parameter $A$ falls to a lower permanent level $A' < A$. Let us focus on the evolution of the capital stock $k_t$ and its price $q_t$, shown in Figure 13.3. The drop in $A$ shifts the $\dot{q} = 0$ schedule to the left. The other locus is unaffected. The steady state of the economy also moves left. The new steady state capital stock is $k^{*\prime} < k^*$.

Dynamic adjustment is as follows. $q$ falls on impact all the way to the saddle path that goes through the new steady state. Over time the system moves along the new saddle path until it reaches the new steady state. Along this trajectory the rate of investment is negative, eventually hitting $\dot{k}_t = 0$ as the economy lands on the new steady state. Notice that during the transition path the price of the asset *increases*. Why would it increase if the productivity of capital has fallen? Remember that at all times the asset has to deliver the opportunity cost of capital ($r$), if it does not you should sell the asset (this is the reason its price falls). While the economy adjusts to its lower capital stock, the marginal product of capital is lower. Thus, you should expect a capital gain if you are to hold the asset. The price initially drops sufficiently to generate that capital gain.

**Figure 13.2 The dynamics of investment**

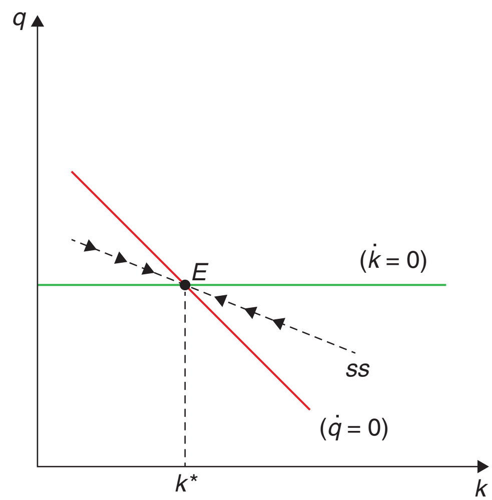
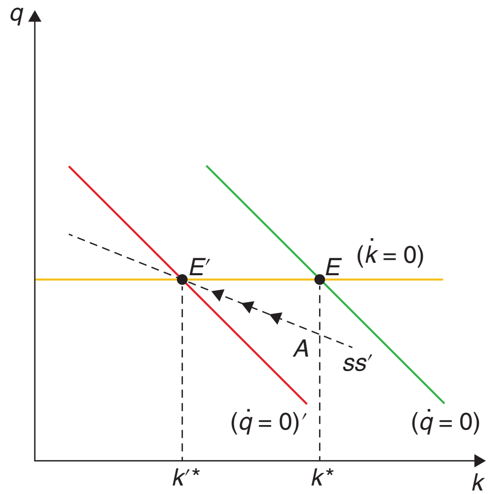


**Figure 13.3 The adjustment of investment**




### 13.2.4 | The role of $\chi$

We can see from the equations for the steady state that the parameter that indicates the costliness of adjustment does not matter for $k^*$ or $y^*$. But it does matter for the dynamics of adjustment.

Note that as $\chi$ falls, costs of investment rise, and vice versa. In the limit, as $\chi$ goes to zero, capital never changes; we see from (13.20) that, in this, case $\dot{k}_t = 0$ always.

Consider what would happen to the reaction of capital and its price to the previous shock as $\chi$ rises. One can show (solving the model explicitly) that the higher $\chi$, the lower the cost of adjustment and the flatter the saddle path. Intuitively, since with high $\chi$ adjustment will be fairly cheap and, therefore, speedy, the price of capital $q$ does not have to jump by a lot to clear the market.

In the limit, as $\chi$ goes to infinity, adjustment is costless and instantaneous. The capital stock is no longer a sticky variable, and becomes a jumpy variable. Therefore, its price is always 1, and the system moves horizontally (along $\dot{k} = 0$) in response to any shocks that call for a different steady state capital stock.

## 13.3 | Investment in the open economy

In a small open economy, introducing a smooth adjustment of investment implies that the current account will change as a result of shocks. A somewhat counterintuitive result is that a negative productivity shock will lead to a surplus in the current account, as capital will fall gradually and consumption anticipates the future decline.

We can now embed the investment behaviour of the firm in a small open economy framework of Chapter 4. In other words, we revisit the open-economy Ramsey model that we have seen before, but now with the more realistic feature of adjustment costs. We want to understand how investment interacts with consumption and savings and how it matters for the trade balance and the current account. We will see that the assumption of adjustment costs has important implications, particularly for the latter.

Consider a small open economy perfectly integrated with the rest of the world in both capital and goods markets. There are two assets: the international bond and domestic capital, just as we have seen before.

In the small open economy, there is a representative consumer and a representative firm. The former can invest in international bonds or in shares of the firm. In fact the consumer owns all the shares of the firm, and receives all of its profits.

### 13.3.1 | The consumer's problem

The utility function is

$$\int_0^\infty u(c_t)e^{-\rho t}dt, \tag{13.30}$$

where $c_t$ denotes consumption of the only traded good and $\rho(>0)$ is the rate of time preference. We assume no population growth.
The consumer's flow budget constraint is

$$\dot{b}_t = rb_t + \pi_t - c_t, \tag{13.31}$$

where $b_t$ is the (net) stock of the internationally-traded bond; $r$ is the (constant and exogenously-given) world real interest rate; and $\pi_t$ is firm profits. Notice that the consumer is small and, therefore, takes the whole sequence of profits as given when maximising his utility.

Notice that the LHS of the budget constraint is also the economy's current account: the excess of national income (broadly defined) over national consumption.
Finally, the solvency (No-Ponzi game) condition is

$$\lim_{T\to\infty} b_T e^{-rT} = 0. \tag{13.32}$$

The Hamiltonian can be written as

$$H = u\left(c_t\right) + \lambda_t\left[rb_t + \pi_t - c_t\right], \tag{13.33}$$

where $\lambda_t$ is the costate corresponding to the state $b_t$, while control variables is $c_t$.

The first order condition with respect to the control variables is

$$u'(c_t) = \lambda_t. \tag{13.34}$$

The law of motion for the costate is

$$\dot{\lambda}_t = \lambda_t(\rho - r) = 0, \tag{13.35}$$

where the second equality comes from the fact that, as usual, we assume $r = \rho$.

Since $\lambda$ cannot jump in response to anticipated events, equations (13.34) and (13.35) together say that the path of consumption will be flat over time. In other words, *consumption is perfectly smoothed over time*. Along a perfect foresight path the constant value of $c_t$ is given by

$$c_0 = c_t = rb_0 + r\int_0^\infty \pi_t e^{-rt}dt, \qquad t \geq 0. \tag{13.36}$$

Consumption equals permanent income, defined as the annuity value of the present discounted value of available resources.

Notice that the second term on the RHS of (13.36) is the present discounted value of profits, which is exactly the object the firm tries to maximise. Hence, by maximising that, the firm maximises the feasible level of the consumer's consumption, and (indirectly) utility.

### 13.3.2 | Bringing in the firm

We now need to solve for the path of investment and output. But we have done that already, when we solved the problem of the firm. We know that the firm's behaviour can be summarised by the initial condition $k_0 > 0$, plus the pair of differential equations

$$\dot{k}_t = \chi \left(q_t - 1\right) k_t. \tag{13.37}$$

and

$$\dot{q}_t = rq_t - Af'\left(k_t\right) - \frac{1}{2}\chi \left(q_t - 1\right)^2. \tag{13.38}$$

Recall that (13.37) and (13.38) are a system of two differential equations in two unknowns, which can be solved independently of the other equations in the system.

Recall also that profits are defined as

$$\pi_t = Af\left(k_t\right) - \psi\left(\dot{k}_t, k_t\right) - \dot{k}_t. \tag{13.39}$$

- Once we have a solution (13.37) and (13.38), we know what $k_t$, $\dot{k}_t$, $q_t$ and $\dot{q}_t$ are.
- With that information in hand, we can use (13.39) to figure out what the whole path for profits $\pi_t$ will be.
- Knowing that, we can use equation (13.36) to solve for consumption levels.
- Knowing output levels $Af\left(k_t\right)$, investment rates $\dot{k}_t$, and consumption levels $c_0$, we can use this information plus the budget constraint (13.31) to figure out what the current account $\dot{b}_t$ is and, therefore, the path of bond-holdings $b_t$.

### 13.3.3 | Initial steady state

Consider the steady state at which debt holdings are $b_0$ and the capital stock is constant. Let stars denote steady state variables. From (13.37), $\dot{k}_t = 0$ implies $q^* = 1$. In turn, setting $\dot{q}_t = 0$ in (13.38) implies $r = Af'(k^*)$, which shows that the marginal product of capital is set equal to the world rate of interest. It follows that, in steady state, output is given by $y^* = Af(k^*)$. Finally, consumption is given by

$$c^* = rb_0 + y^*. \tag{13.40}$$

In the initial steady state, the current account is zero. The trade balance may be positive, zero, or negative depending on the initial level of net foreign assets:

$$TB \equiv y^* - c^* = -rb_0.$$

### 13.3.4 | The surprising effects of productivity shocks

Suppose again that, starting at this steady state, the productivity parameter $A$ falls to a lower permanent level $A' < A$. To make life simpler, suppose that initially bond-holdings are zero ($b_0 = 0$).

Focus first on the evolution of the capital stock $k_t$ and its price $q_t$, shown in Figure 13.3. The drop in $A$ shifts the $\dot{q} = 0$ schedule to the left. The other locus is unaffected. The steady state of the economy also moves left. The new steady state capital stock is $k^{*\prime} < k^*$.

At time zero, $q$ falls on impact all the way to the saddle path that goes through the new steady state. Over time the system moves along the new saddle path until it reaches the new steady state. Along this trajectory the rate of investment is negative, eventually hitting $\dot{k}_t = 0$ as the economy lands on the new steady state.

Notice that net output, defined as $y_t - \dot{k}_t - \psi\left(\dot{k}_t, k_t\right)$, may either go up or down initially, depending on whether the fall in gross output $y_t$ is larger or smaller than the change in costs associated with the decline in investment. Over the long run, however, the effect is unambiguous: Net output is lower, since gross output falls and investment tends to zero. Figure 13.4 shows net output falling initially, and then declining further to its new steady state level.

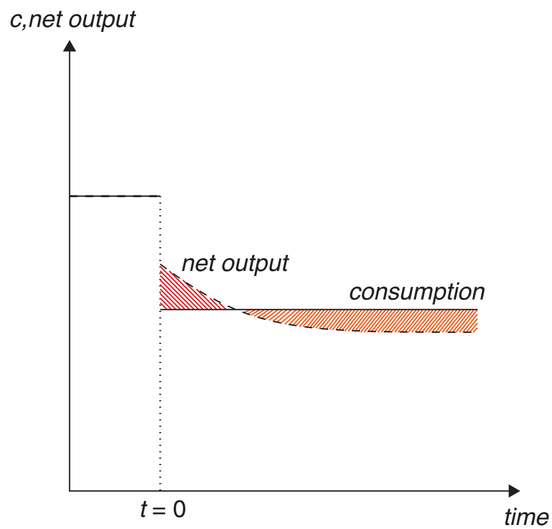
The level of consumption is constant and given by

$$c_0' = r\int_0^{\infty} \left(A'f\left(k_t\right) - \dot{k}_t - \psi\left(\dot{k}_t, k_t\right)\right) e^{-rt}dt < c_0 \qquad t \geq 0. \tag{13.41}$$

Graphically (in terms of Figure 13.4), consumption is determined by the condition that the thatched areas above and below it be equal in present value.

What about the current account? Since initial bonds are zero, having consumption below net output (see Figure 13.4) must imply that the economy is initially running a current account surplus, saving in anticipation of the lower output in the future. In the new steady state, the net foreign asset position of the economy is that of a creditor: $b^{*\prime} > 0$ and the current account goes back to zero.

**Figure 13.4 The effect on the current account**



This result led to a novel and surprising interpretation of the current account. Imagine, for example, a fall in productivity that arises from an oil crisis (such as that in the 1970s), affecting an oil importing country.[6] Figure 13.4 suggests that this shock will lead to a surplus in the current account. Notice that this may challenge your intuition; if oil has become more expensive and this is an oil importing country, shouldn't the current account deteriorate? Why is this wrong? It is wrong because it fails to take into account the adjustment of aggregate spending in response to the shock. The oil shock not only makes imports more expensive, it lowers expected future income. As consumption smooths future decreases in income, it reacts strongly in the short run, in fact, ahead of the output decrease, leading to a surplus. By the same token, if you are an oil-exporting country, a positive shock to the price of oil would stimulate consumption and investment. As output will increase over time the reaction of consumption can be strong, leading to a deficit. Of course this result depends on a number or assumptions, for example that the shock be permanent and believed to be so. Changing any of these assumptions can change this result.

The bottom line is that these intuitions that are aided by our use of a general equilibrium format. In fact, when Sachs et al. (1981) tested these results by looking at the response of the current account to the oil shocks of the 1970's they found that the results of the theory held surprisingly well in the data. Oil exporting countries quickly (though not instantaneously) were experiencing deficits, while oil importing countries managed to show surpluses.

On a final note, you may recall that the adjustment process is quite different from what we had in the open-economy Ramsey model without adjustment costs in response to a permanent shock. There the current account of an economy in which $r = \rho$ was always zero. This was because all adjustments could be done instantaneously – if the shock led the economy to reduce its stock of capital, it could do so immediately by lending abroad. Now, because of adjustment costs, this process takes place over time, and we have current account dynamics in the transition.

## 13.4 | What next?

If you are interested in understanding better the investment process, a good starting point is, again, the Dixit and Pindyck (1994) masterpiece *Investment under Uncertainty*. The Bloom et al. (2007) paper provides a wonderful reference list with the most important pieces written to that date. In more recent years a debate has ensued on the nature of investment in an increasingly digitalised world where firms do not need so much capital as they do human capital. Crouzet and Eberly (2019) is a good entry point into this debate.

## Notes

1 The classical reference on this issue is Dixit and Pindyck (1994). See, for example, Brennan and Schwartz (1985) for mining projects. More recently, Schwartz has found that new reserves additions to oil companies *reduce* the value of the companies. In this case, they are an option to a loss! (See Atanasova and Schwartz (2019)).

2 This issue has a long pedrigee in economics. For a relatively recent reference, with many references thereof, you may want to check Bloom (2009).

3 Careful with the use of the machines imagery! It brings to mind right away the role of indivisibilities – you can't purchase 3.36 machines – while we will assume that investment is perfectly divisible. Indivisibilities will bring in other issues that are beyond our present scope.

[4] We refer rather loosely here to profit, but really the equation depicts the free cash flow that the investment project will generate over its lifetime.

[5] How can we show that? Note that (13.20) immediately implies that, for $\frac{\dot{k}}{k}$ to be constant, we must have $q$ constant. Using (13.21) to obtain $\frac{\dot{q}}{q}$, and using the fact that $q$ is constant, we see that we must have the ratio $\frac{f'(k)}{q}$ constant, which requires that $k$ is also constant.

[6] One way to think about this is by thinking of oil as an input that has become more costly, akin to a fall in labour or capital productivity.

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