Annex B: Further explanation of the Combinatorial Clock Auction (CCA) format and bidding in the UK's 2013 auction

The CCA format, described in Chapter 8, includes some complex features. This annex provides further explanation of key aspects of the design, illustrated by practical examples from the UK's CCA in 2013: how bidding in a CCA operates; the second-price rules used to set principal stage prices, which have also been adopted for assignment stages in auctions using generic lots in the UK and elsewhere; and the CCA's complicated activity rules.

B1 Bidding in the CCA format

In a CCA the participating firms make their package bids in two component stages – the clock rounds followed by the supplementary bids round, which together comprise the principal stage determining the amount of each band of spectrum won by the bidders. After receiving all the principal stage bids made by the operators, the regulator then works out the winning combination of packages. I go through each of these three parts of the CCA format (clock rounds, supplementary bids round, and computation of the winners), using bids from the 2013 auction to explain the nature of package bidding, to show bids that incorporated synergy values, and to demonstrate the risk of surprise outcomes for the operators.

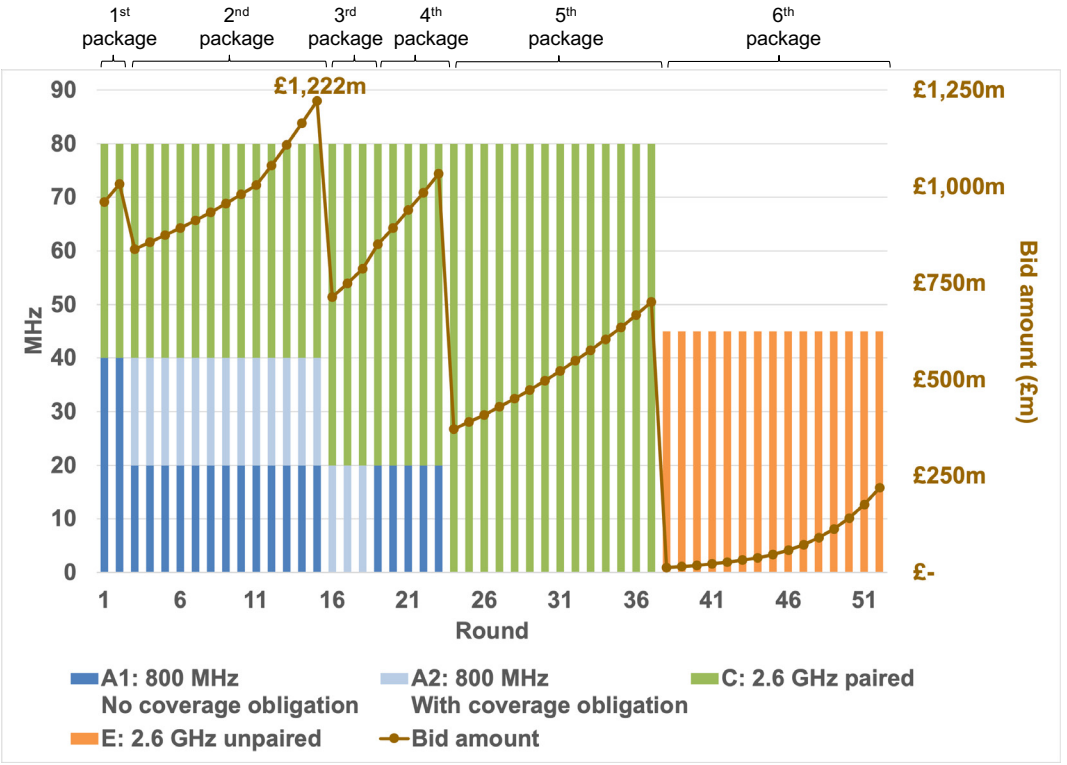
There were four main lot categories included in the 2013 auction:

- For the coverage spectrum in the 800 MHz band, one category (labelled A2) was for a single lot of 20 MHz with an attached coverage obligation, and a second category contained four lots of 10 MHz each without any coverage obligation (A1).
- For the capacity spectrum in the 2.6 GHz bands, paired spectrum was in 14 lots of 10 MHz (category C) and unpaired spectrum was in nine lots of 5 MHz (category E).[1]

Clock stage

To illustrate the nature of package bids in the clock stage of a CCA, Figure B1.1 shows the bids in 2013 by one of the operators, EE, for six different packages of spectrum across the 52 clock rounds. The stacked bars in the columns for each round show the mix of bands that EE sought, and the bid amounts are shown by the brown line. In the first two rounds, EE bid for 40 MHz in each of the 800 MHz and 2.6 GHz paired bands (categories A1 and C). In rounds 3 to 15 it bid for the same spectrum amounts, but switched half of its 800 MHz demand to the spectrum with a coverage obligation (category A2). Its largest clock bid amount of £1,222 million was in round 15. Then in rounds 16 to 37, EE bid for the same total amount of spectrum, but switched the package composition from 800 MHz to more of the lower-priced 2.6 GHz paired in each of rounds 16 and 24. It also switched in round 19 between 800 MHz with and without a coverage obligation. From round 38 EE made another switch

Figure B1.1. Package bids by EE in each clock round in the 2013 auction



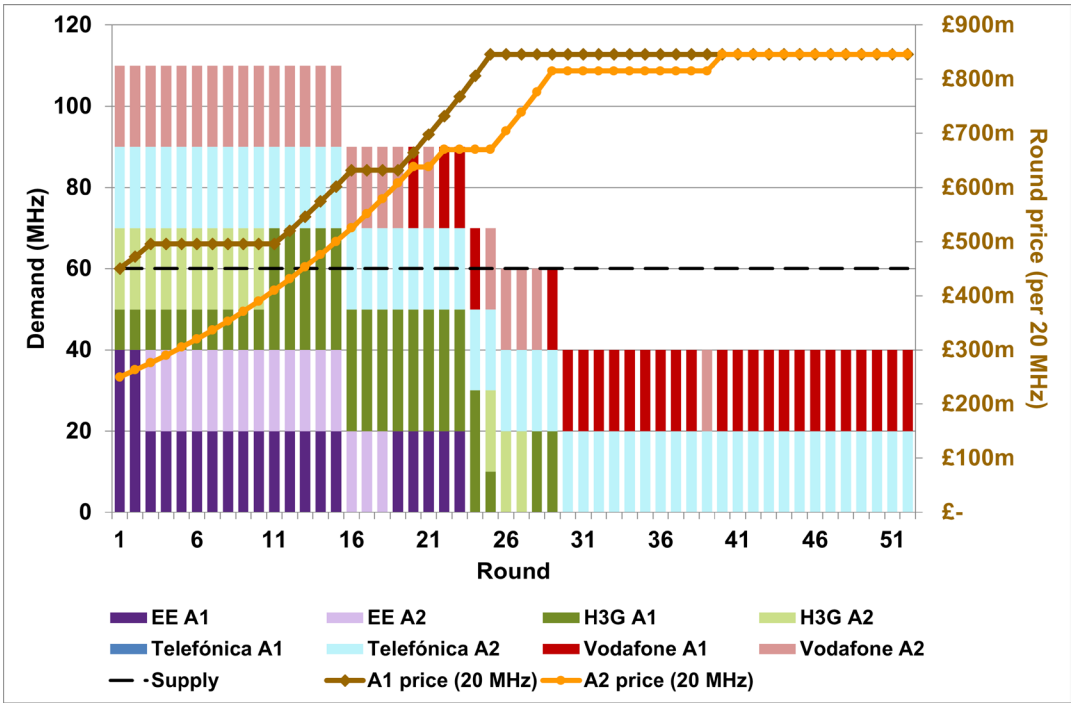
Source: Author from Ofcom auction documents.
Note: EE's largest bid amount is shown rounded to the nearest £ million.

to a smaller amount of 2.6 GHz unpaired spectrum (category E). Overall, EE's different packages displayed just one change in the amount of spectrum, but major changes across the rounds in the package composition by band.

The progress in the clock stage in a CCA depends on the package bids from all bidders. When there is excess demand in any category, the price is increased in the next round. In the 2013 auction, the round price increases are reflected in the rising segments for EE's bid amount line in Figure B1.1 (before EE reduced its bid amounts by switching several times to packages with lower-priced spectrum).

The clock stage in a CCA ends when there is no excess demand in any category. In the 2013 auction there were 52 clock rounds. Starting with the two 800 MHz categories, it took 40 rounds for aggregate demand to settle at a level of demand that was less than supply, as shown in Figure B1.2. The colours signify different bidders, and the lighter and darker shading indicates each operator's bids for 800 MHz with the coverage obligation (A2, shown lighter) and without it (A1, in the darker shade). Some bidders switched between these categories depending on their relative prices, such as Vodafone shown in red. Telefónica set out its stall to win the single A2 lot with the coverage obligation, bidding for it in every round (the pale blue blocks). EE in purple did not bid aggressively for 800 MHz spectrum, and dropped out of the band in round 24. H3G in green only made bids in this band until round 29.

Figure B1.2. Clock bids in the 800 MHz band in 2013, showing the evolution of bids and switching between categories A1 and A2



Source: Author from Ofcom auction documents.

The aggregate demand for the 800 MHz band was the same as supply at 60 MHz as early as round 26. But bidding continued because in a simultaneous auction nothing is resolved until all categories are finished, and there was still excess demand for some of the spectrum. For example, the 800 MHz demand in round 26 was composed of no demand in category A1 but excess demand for the one A2 coverage lot due to bids for three lots by H3G, Telefónica, and Vodafone (shown by the paler shades for these bidders). So the price of A2 went up in the next few rounds (see the upward sloping A2 price line in orange). In response two bidders, H3G and Vodafone, switched their demand from A2 to A1. By round 29 there was no excess demand in either A1 or A2, leading to no price increases (both price lines were flat). Bidding for 800 MHz remained open because there was still excess demand in the 2.6 GHz categories, and there were a few further changes. H3G dropped out of the band in round 30, leading to excess supply of 20 MHz in A1. Vodafone switched from A1 to A2 in round 39, leading to excess demand in A2 and so a price increase in the next round that made the per MHz price of A2 as high as A1. In round 40 Vodafone switched back to A1 so that only Telefónica was bidding for A2 and *excess supply* in A1 was again at 20 MHz. There were no changes in bids or prices in the 800 MHz categories after that.

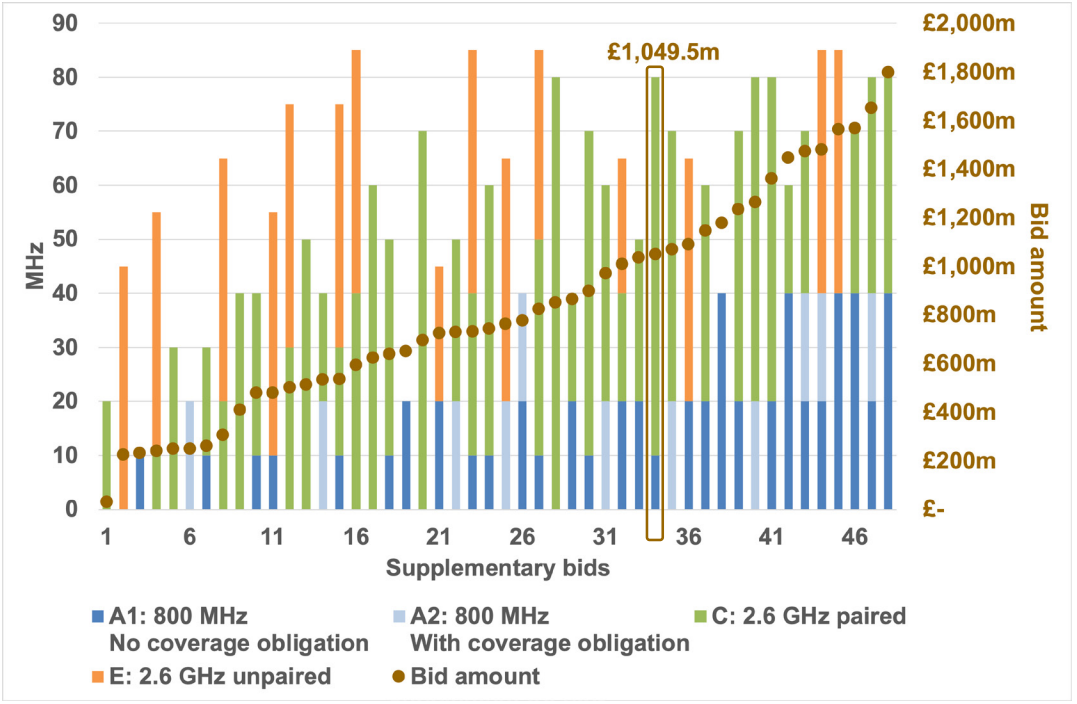
However, there was still excess demand in the 2.6 GHz categories. In 2.6 GHz paired (C), aggregate demand across five bidders was more than double the supply until round 30. It then reduced as the price increased and the last bid change was in round 41, also with a small *shortfall* of demand below

supply of 10 MHz. In 2.6 GHz unpaired (E), all seven bidders were active at some point – the two smaller bidders only made bids in this band and they dropped out in clock rounds 32 (MLL) and 46 (Hong Kong Telecom). This was the lowest-value band in the auction but the last to settle in round 52. The duration of bidding in this category was in large part due to starting at a very low reserve price, only £0.1 million per 5 MHz lot. Demand was initially more than four times larger than supply, and the clock price rose to £24.4 million when the clock stage ended (see Figure B3.3 in the final section of the annex).

Supplementary bids and synergy values

After the clock rounds the next stage of the CCA format is the supplementary bids round, when each bidder can make a large number of bids for mutually exclusive packages. Examples of an operator's supplementary bids are EE's bids for 48 packages in 2013 set out in Figure B1.3, including different quantities and composition of the three spectrum bands (800 MHz in blue, 2.6 GHz paired in green, and 2.6 GHz unpaired in orange) and bid amounts (shown by the brown dots). This number of package bids was more than four of the bidders – HKT and MLL (9 each), Telefónica (11), and H3G (17) – but much less than two other companies – BT (89) and Vodafone (94). In their supplementary bids operators can choose to bid for *additional packages* compared to the clock rounds. EE did so in 2013,

Figure B1.3. Package bids by EE in the supplementary bids round in 2013



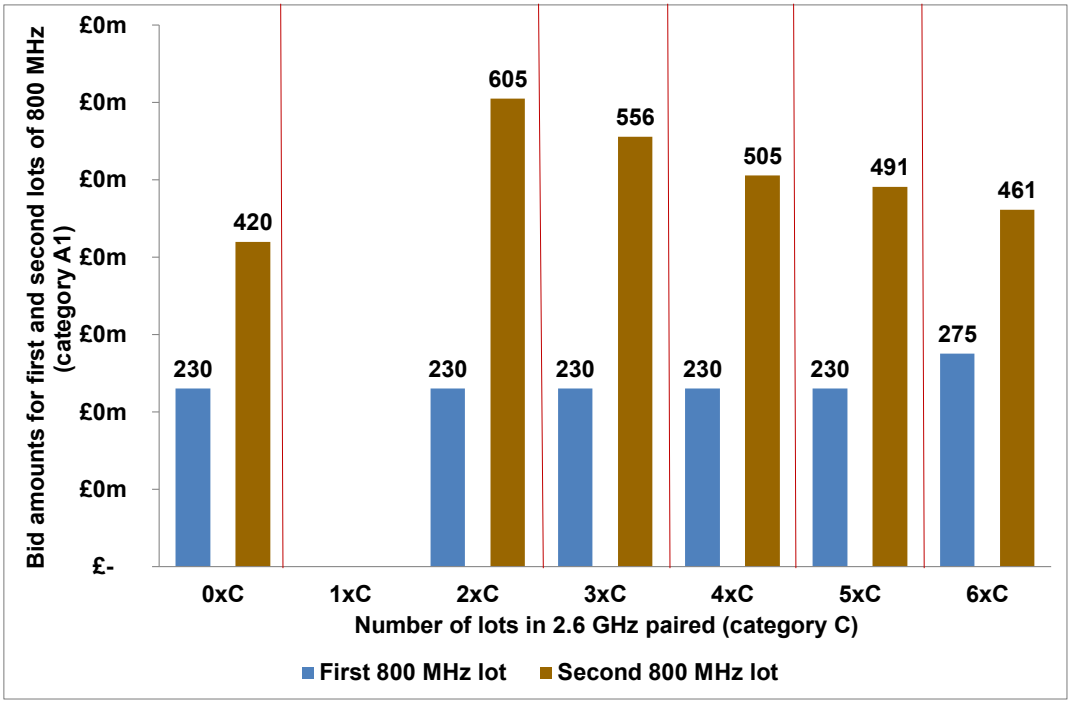
Source: Author from Ofcom auction documents.
Note: EE's winning bid amount is shown rounded to the first decimal point in £ million.

bidding for 42 further packages, including its winning bid which was not one of its six packages in the clock stage. EE's winning bid is highlighted, shown as supplementary bid 34 when arranged by bid amount as in Figure B1.3, namely £1,049.5 million for 10 MHz of 800 MHz and 70 MHz of 2.6 GHz paired (1 lot of category A1 and 7 lots of category C).

One of the advantages of package bidding in a CCA is that it enables operators to express *synergies* in their spectrum valuations. Some of the *within-band* synergies in EE's supplementary bids in 2013 are illustrated in Figure B1.4, showing pairs of incremental bid values for each of the first and second lots of 800 MHz (A1) in packages with different numbers of lots of 2.6 GHz paired (C) shown along the horizontal axis. The incremental bid values for the second 800 MHz lot (the brown bars) are much larger in every case than for the first lot (the blue bars). This rising value per MHz reflects the synergy values. For example, the largest synergy was in packages with two lots of 2.6 GHz paired (2xC):

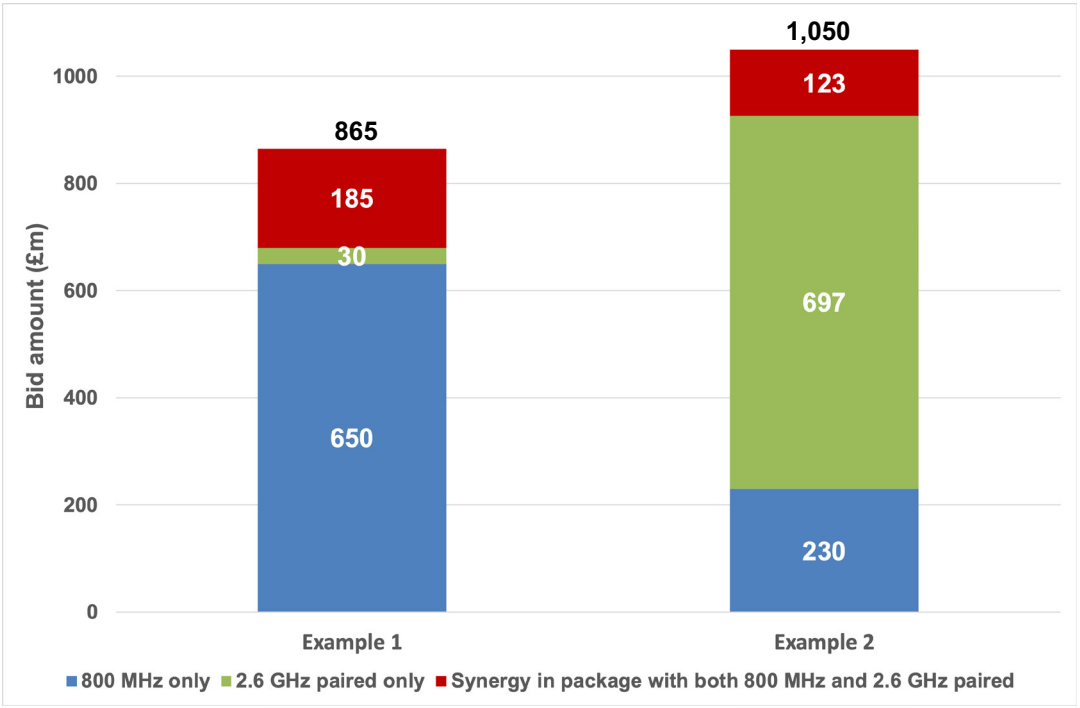
- EE bid £30 million for a package of 2xC only and £260 million for a larger multi-band package also including one lot of 800 MHz (1xA1 plus 2xC). Therefore, its incremental bid value for the first 800 MHz lot was £230 million, the amount shown in the blue bar in the second pair of incremental bid values in Figure B1.4 in the columns labelled 2xC.

Figure B1.4. Examples of within-band synergies in EE's bids in 2013, shown by rising incremental bid values between its first and second 800 MHz lots (category A1) in packages including different amounts of 2.6 GHz paired lots (category C)



Source: Author from Ofcom auction documents.
Notes: The columns for 1xC are blank because EE did not make bids for packages with 1xC.
Incremental bid values are shown rounded to the nearest £ million.

Figure B1.5. Examples of cross-band synergies in EE's package bids in 2013 for combinations of 800 MHz and 2.6 GHz paired spectrum



Source: Author from Ofcom auction documents.
Note: Bid values are shown rounded to the nearest £ million.

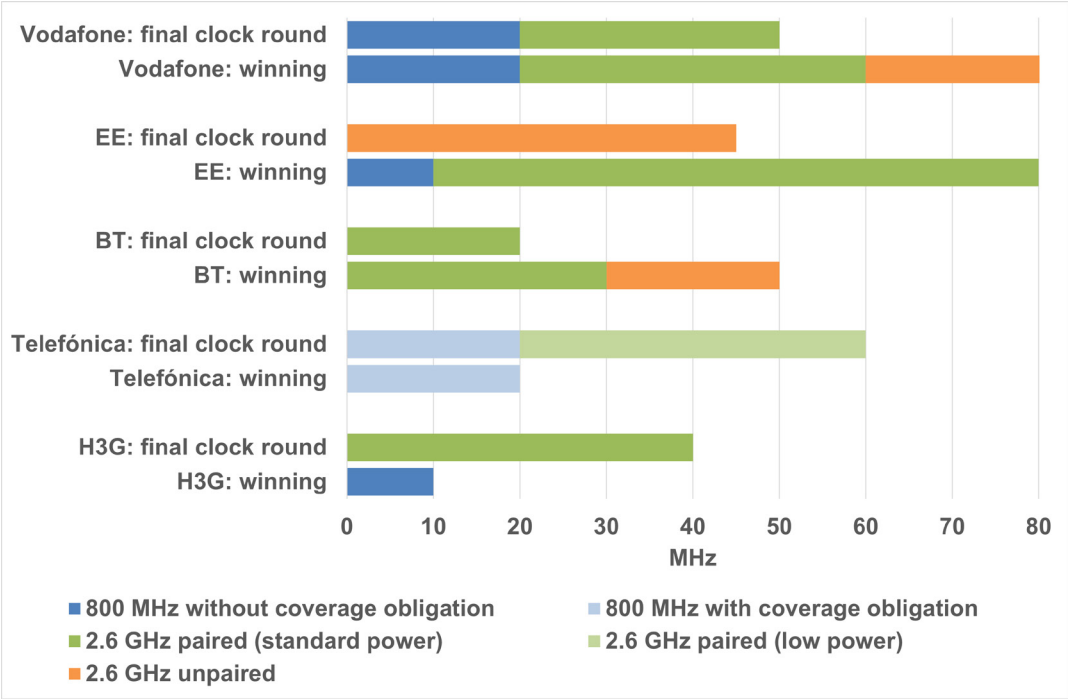
- It also bid £865 million for the larger package which included a second lot of 800 MHz, 2xA1 plus 2xC, at a much bigger incremental bid value for the second 800 MHz lot shown in the brown bar of £605 million (£865 million less £260 million).

Many of EE's bids also displayed *cross-band* synergies, of which two examples are shown in Figure B1.5. EE bid £650 million shown in the blue segment for a package of 800 MHz only (two lots, 2xA1), £30 million in the green segment for a package of 2.6 GHz paired only (two lots, 2xC), but £865 million for the larger package including both these amounts of 800 MHz and 2.6 GHz paired (2xA1 plus 2xC). This exceeded the sum of bids for the constituent smaller packages by the cross-band synergy of £185 million shown in the red segment (21 per cent of the bid of £865 million). The second example relates to EE's winning bid in the auction: it bid £230 million for 800 MHz only (in this case, for just one lot, 1xA1), £697 million for 2.6 GHz paired only (seven lots, 7xC), and £1,050 million for the larger package including both (1xA1 plus 7xC). This included a cross-band synergy of £123 million (12 per cent of £1,050 million).

Spectrum allocation outcome

The spectrum allocated to each operator in the CCA format is identified by an algorithm searching for the set of packages with the highest bid value among all combinations that include no more than

Figure B1.6. Comparison of winning and final clock packages of the five successful bidders in 2013



Source: Author from Ofcom auction documents.

Note: Telefónica's final clock package included a bid for a concurrent low-power licence for a portion of the 2.6 GHz paired spectrum. Operators are shown in descending order of the spectrum amounts in their winning packages.

one package bid from each bidder and avoid exceeding the available spectrum supply. In the case of the 2013 auction, all winning packages after the supplementary bids round differed from those in the final clock round, as set out in the comparison by bidder in Figure B1.6. For each operator, the package of spectrum at the end of the clock stage is shown as its upper stacked bar, and the operator's winning package as its lower stacked bar. Overall, the winning packages were larger, because operators made supplementary bids for the spectrum in excess supply at the end of the clock rounds in the 800 MHz and 2.6 GHz paired bands. However, the changes in the packages were much more extensive than just adding this spectrum to bidders' final clock round packages. Vodafone, EE, and BT won much larger amounts of spectrum than in their packages at the end of the clock stage, and Telefónica's and H3G's winning packages were significantly smaller. EE and H3G also won spectrum in entirely different bands.

A key function of the clock stage of a CCA is to assist the bidders through feedback from the auction providing price and package discovery. However, in practice in 2013 the large changes in packages between the end of the clock stage and the final outcome highlighted the limited usefulness of *package* discovery during the clock stage of that auction. In addition, the limited *price* discovery in the clock stage in 2013 is indicated by the difference in band prices in Figure B1.7 between the prices in the final clock round, and the much lower principal stage prices for the winning packages after the supplementary bids, which were based on the highest losing bids – see the next section of this annex.[2]

Some of the reasons for the limited package and price discovery in 2013 were because of the choice of activity rules – see the final section of the annex.

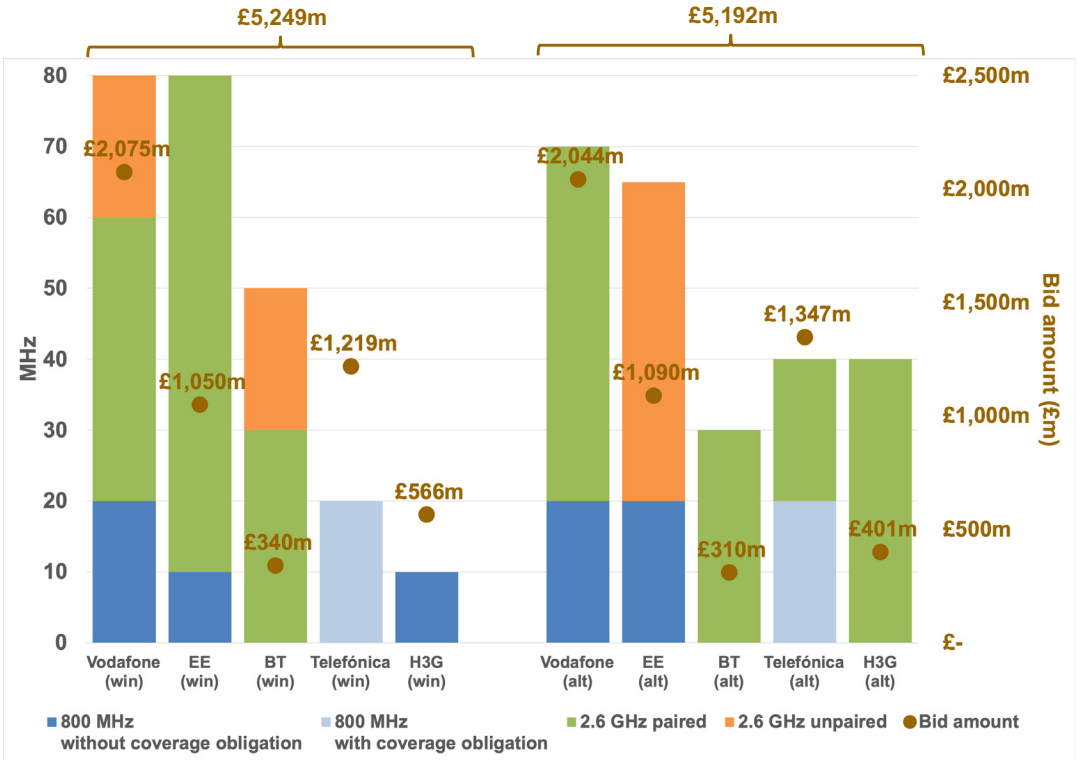
When determining the winning set of packages in a CCA, alternative candidate combinations can involve large not marginal differences in spectrum allocations between bidders. An illustration is two candidate outcomes in the 2013 auction and their bid amounts shown in Figure B1.8, labelled 'win'

Figure B1.7. Comparison of prices in £ million per MHz in the final clock round and for the winning packages after supplementary bids in 2013

	800 MHz	2.6 GHz	
		Paired	Unpaired
Prices in the final clock round	42.3	9.2	2.4
Principal stage prices for the winning packages*	26.9	5.0	1.3

Source: Author from Ofcom auction documents.
Notes: * Principal stage prices (after supplementary bids) were set by package, with no uniquely correct way to decompose them by band. The figure shown for each band is the 'linear reference price' per MHz.[3]
Prices are shown rounded to the first decimal point in £ million.

Figure B1.8. Comparison of the winning combination of package bids and an alternative combination in 2013



Source: Author from Ofcom auction documents.
Note: Bid amounts are shown rounded to the nearest £ million. Operators are shown in descending order of the spectrum amounts in their winning packages.

and 'alt' – these were the winning combination and the losing combination for the choice of alternative spectrum floors (reserved spectrum) obtained by H3G (see Section 10.1). Each combination includes one package bid from the five winning operators, and both would have sold all the spectrum. The winning combination on the left-hand side had total bid value of £5,249 million. The total bid value of the alternative on the right-hand side was only 1 per cent less at £5,192 million. In order to win, the total bid value in the alternative combination would have needed to be slightly higher to exceed the winning combination. If so, the spectrum allocation would have been very different: H3G with 40 MHz instead of 10 MHz and in a different band – 2.6 GHz paired instead of 800 MHz; Telefónica with 20 MHz more by adding 2.6 GHz paired spectrum to the 800 MHz with the coverage obligation; and Vodafone, EE, and BT with less spectrum in their packages and also a different composition of bands, such as EE having 10 MHz more of the 800 MHz band but only 45 MHz of the 2.6 GHz unpaired spectrum instead of 70 MHz in the 2.6 GHz paired band. The large differences between these two candidate outcomes emphasised the scope for *surprise outcomes* in the CCA.

B2 Second-price rules

After identifying the winning set of packages in the CCA, the regulator then determines the prices to be paid by the successful bidders, using information from the highest losing bids. To explain the specific version of the second-price rules used in the CCA format, I build up the logic from individual to collective opportunity cost along with examples, both illustrative and from the 2013 auction. The same pricing rule is often also adopted to set the prices in assignment stages, as in the UK in 2013, 2018, and 2021.

Vickrey prices: individual opportunity cost

The format chosen for the assignment stage in UK auctions and elsewhere in many cases is a sealed-bid, combinatorial, second-price auction for each band. This is a simpler version of the supplementary bids round of the CCA format. Both are related to a 'Vickrey auction' (named after its inventor, the Nobel Memorial Prize winner William Vickrey).[4] It has desirable theoretical properties of providing incentives for straightforward bidding, because the prices paid by the winners are set by bids made by other bidders (highest losing bids) and so are independent of a firm's own bids. Vickrey prices are set at individual opportunity cost, the lowest bid a winning operator could make before it fails to remain a winner because of the highest losing bids. An equivalent way to express this price is that each winning bidder obtains a discount on its winning bid (the 'Vickrey discount') that is equal to the value it brings to the auction (which in turn is given by the difference in total bid value with and without the bidder). However, Vickrey auctions also have well-known deficiencies.[5] For example, they can result in very low prices, and are open to gaming such as collusion.[6]

A simple example of bidder valuations is shown in Figure B2.1 for three bidders, called Camel, Dolphin, and Elephant. Dolphin and Elephant view the two items, A and B, as being close substitutes, so that they have almost the same value of 24 or 25 for each, and they gain no extra value from winning both. For Camel the two items are pure complements with no stand-alone value for either individually but a large synergy of 25 from winning both.

With straightforward bidding of these values, the highest total value of 50 and auction efficiency is achieved, with Dolphin winning A and Elephant winning B. However, the Vickrey price for both is zero, even though the losing bidder, Camel, bid 25 for A and B. We can see this zero price from either of the ways to derive the Vickrey prices, taking Dolphin's price as an example:

Figure B2.1. Simple example of values with substitutes and complements

	Items bid for		
Bidders	Item A	Item B	Package of A and B
Camel	0	0	25
Dolphin	**25**	24	25
Elephant	24	**25**	25

Source: Author.
Note: The optimal winners of the items for auction efficiency are highlighted.

- The lowest bid Dolphin could make before failing to win item A is zero, given that Elephant makes a bid of 25 for B which is the same as Camel's bid for the package of A plus B.
- The Vickrey discount is 25 (the total bid value with Dolphin of 50 less the total value without it of 25). So Dolphin's winning bid of 25 less this discount is zero.

The example illustrates the potential problem of low revenue with Vickrey prices, which could be exploited by bidder collusion, or by 'shill' bidding where Dolphin and Elephant are really only one company but pretended in the auction to be two different bidders. The existence of large synergies for Camel means that a losing bidder placed bids in the auction that are higher than the Vickrey prices to be paid by the winners (or more generally, it could be a coalition of losing bidders that placed the higher bids). If so, the prices lie outside the 'core', the set of outcomes that involve no coalitions preferring an alternative (see Section 3.2).

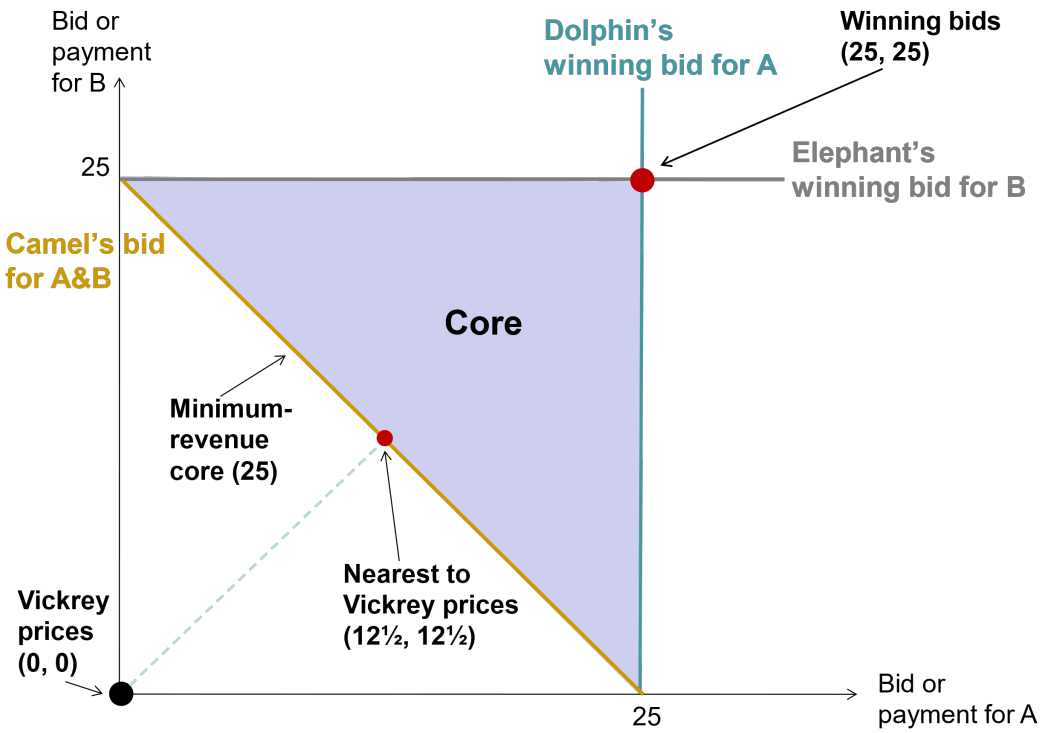
Core prices: collective opportunity cost

Some of the disadvantages of Vickrey auctions are alleviated by a modified second-price rule, requiring prices to have the following features:

- Prices lie inside the core, so that no losing bidder expressed through its bids that it was willing to pay more than the auction price for items won by others.[7]
- They represent the lowest prices in the core – minimum-revenue or 'bidder optimal' core prices.[8] The reason for this label is that the pricing rule makes bidders as well off as possible given their bids, while keeping prices above the level that losing bidders were willing to pay.
- Since the minimum-revenue core can include many sets of prices which are consistent with the winning packages and higher than the losing bids, a 'reference rule' is needed to determine which is used.[9] Although many are possible in principle, the reference rule that has been used in practice in the UK and elsewhere is 'nearest-to-Vickrey' prices.[10]

Figure B2.2 illustrates these features for the simple example. The core is the triangular shaded area bounded by each of the winning bids of Dolphin and Elephant, and the highest losing bid by Camel. The minimum-revenue core of 25 is shown as the set of prices on the line forming the bottom diagonal edge of the triangle (reflecting Camel's bid). The Vickrey prices of (0, 0) lie outside the core due to synergies in Camel's bid. The prices that are nearest to Vickrey prices in the minimum-revenue core are (12½, 12½). There is also a large gap between the winning bids of (25,25) and the auction prices of (12½, 12½).

Figure B2.2. Simple example of core and Vickrey prices



Source: Author.

Minimum-revenue core prices reflect the collective opportunity cost of allocating all the winning packages to the set of winners (which in the example is 25, the highest losing bid by Camel). Core prices have advantages of stability and fairness, because they were not outbid by any losing bidder or coalition of losing bidders. If revenue-raising is an objective, core prices can also avoid the very low revenue sometimes associated with Vickrey prices. But in principle at least, core prices can also weaken the incentives for straightforward bidding, because it is now theoretically possible for a winning bidder to affect the price it pays through its own bids. However, in practice it may be very difficult to identify how to do this successfully, and it is usually not a risk-free strategy.[11]

In the example, the Vickrey prices are outside the core, which has been the case in some North American auctions.[12] However, it is quite possible that they lie within the core and, if so, they form the unique, minimum-revenue core prices. In practice, Vickrey prices have been in the core in all UK auctions up to 2021 which have used the core pricing rule for either assignment stages or the principal stage in the CCA format.

Practical experience setting prices

An example of a *principal stage* price in the 2013 auction is for BT, which won a package of three lots of 2.6 GHz paired and four lots of 2.6 GHz unpaired (3xC plus 4xE). In the absence of BT's bids, the spectrum in its winning package would have been won by other winning bidders increasing the sizes of their winning packages. These represented the relevant highest losing bids to determine BT's

package price. The sum of these incremental bid values for increasing their winning packages was BT's price of £186 million:

- Telefónica would have won an additional 2xC at an incremental bid value of £128 million compared to its winning package.
- Vodafone would have won an additional 1xC and 4xE at an incremental bid value of £58 million.

The other way to derive this Vickrey price was BT's winning bid of £340 million less its Vickrey discount of £154 million. The Vickrey discount was derived as the value that BT brought to the auction: the total bid value in the winning combination of £5,249.5 million less the highest total bid value without BT of £5,095.5 million.

The derivation of other winners' Vickrey prices was more complicated (apart from H3G which paid the reserve price for the spectrum floor it won). In the absence of their bids, there would have been a rearrangement of packages, not just a simple increase in the size of winning packages (as applied to BT's price). An example is the relevant highest losing bids to determine Vodafone's price of £791 million for a package of spectrum in three bands: two lots of 800 MHz, four lots of 2.6 GHz paired, and five lots of 2.6 GHz unpaired. The eight components of Vodafone's Vickrey price included losing bids from all six other bidders in the auction plus the reserve prices for 800 MHz and 2.6 GHz paired spectrum.[13]

Assignment stage prices are usually simpler to derive than principal stage prices, with practical examples set out in Annex A. In the assignment stage, each bidder can only win a single frequency location in each band, such as top, middle, or bottom of the band. The stakes are also lower because the values of different frequency locations are usually a modest fraction of the values of the spectrum (although there are exceptions). The practical bottom line is that, regardless of the intricacies of the underlying theory and its implementation, assignment stages with this format and pricing rule seem to work well.[14]

B3 Activity rules

CCA activity rules apply to bids in both the clock stage and the supplementary bids round (see Section 8.4). As an example I explain the rules which the regulator decided to apply in the 2013 auction, illustrated using some of the bids in 2013. I also outline other possible activity rules which have been deployed elsewhere, such as in Ireland's CCA auctions.[15]

Non-increasing demand in the clock stage as prices rise

The first element of the activity rules in 2013 prevented a bidder from increasing its quantity demanded as prices rose in the clock stage of the auction (similar to the activity rule generally used in SMRAs). This rule was intended to encourage the operator to bid more straightforwardly in early rounds and not to hide its demand. It applied during the clock stage using eligibility points set by the regulator for spectrum in different frequency bands. Each operator had to decide its initial number of eligibility points at the start of the auction (and pay a deposit to the regulator accordingly at £1,000 per eligibility point). The operator's eligibility points could not increase during the auction, but only decrease

monotonically (i.e. either stay the same or fall) in line with the quantities of spectrum it bid for in each round. For example, in clock round 16, EE switched some of its demand from 800 MHz to the 2.6 GHz band with much lower associated eligibility than its package in round 15 (see Figure B1.1), which resulted in an irreversible reduction in its eligibility (from 9,600 to 5,400 points).

However, the monotonic eligibility rule can provide incentives for an operator to depart from straightforward bidding so as to preserve its eligibility – by bidding on larger packages with more eligibility points it could gain greater flexibility in future rounds, as probably happened in 2013. A rule that could be added to alleviate this incentive, but was not applied in 2013, is the '*relaxed activity*' rule, which permits an operator to bid for a package in the clock stage with more eligibility points than its current level in specified circumstances (where it is consistent with revealed preference, whose meaning is discussed in the next subsection).[16]

Relative cap in the supplementary bids round: consistency with revealed preference

The second element of the 2013 activity rules linked bids in the clock stage and the supplementary bids round in order to encourage straightforward bidding throughout the auction.[17] The operator's supplementary bid for its package in the final clock round was uncapped, so that it could increase the bid amount as high as it wished in the supplementary bids round. For example, Telefónica bid £890 million in the final clock round for a package comprising the 800 MHz coverage obligation lot (A2) and one lot in category D2 (which was for concurrent low-power licences for a portion of the 2.6 GHz paired spectrum), and in its supplementary bids Telefónica increased the bid amount for this package by 40 per cent up to £1,263 million.

However, the 'relative cap' activity rule then placed a limit on a firm's bid amounts for both smaller and larger packages relative to its supplementary bid for the final clock package:[18]

$$C = B + (RP - P)$$

where:

C was the cap on the supplementary bid;
B was the supplementary bid on the constraining package; and
(RP-P) was the difference in clock round prices between the package and the constraining package in the final clock round (for smaller packages), or in the round when eligibility was dropped below the eligibility for the package (for larger packages).

The bid for a *smaller* package was capped relative to the supplementary round bid on its final clock package, with the permitted differential taking account of prices in the final clock round. For example, Telefónica's winning bid in 2013 was £1,219 million for a package just containing the 800 MHz coverage obligation lot, A2. This was a smaller package than its final clock package of A2 plus one lot of D2. The relevant difference in clock round prices was the price of a D2 lot of £44 million in the final clock round, since one D2 lot was the difference in spectrum between the packages. So the relative cap on Telefónica's winning bid for the A2 lot was the supplementary bid for the final clock package £1,263 million less £44 million, or £1,219 million. Telefónica chose to make a bid on its winning package up to the maximum permitted by this relative cap.

Figure B3.1. Example of EE's revealed preference for larger and smaller packages (including A1, A2, and C) from clock stage bids in rounds 15 and 16 in 2013, and consequent relative cap constraint on its supplementary bids

Indicators	A1	A2	C	Package bid amount at prices in:		Supplementary bids for packages
				Clock round when bid was made	Clock round 16	
Round 15: – Number of lots – Clock prices per lot	2 £301m	1 £500m	4 £29.9m	£1,222m	(RP) £1,283m	£1,652m
Round 16: – Number of lots – Clock prices per lot	0 £316m	1 £525m	6 £31.4m	£713m	(P) £713m	£1,264m
Relative cap constraint (RP–P), and difference between EE's supplementary bids					£569m	£388m

Source: Author from Ofcom auction documents.
Note: Bid amounts are shown rounded to the nearest £ million.

The bid for a *larger* package was capped relative to the supplementary round bid on the 'constraining package', which was the one that the operator bid for in the latest clock round when it had sufficient eligibility to bid on the larger package, but chose instead to bid for the constraining package (and so dropped its eligibility below the level of the larger package). The permitted differential in supplementary bids between the larger package and the constraining package reflected the clock prices in the round when eligibility was dropped. The purpose of the relative cap was for bids to be in accordance with *revealed preference* as indicated when the bidder dropped its eligibility in the clock stage by switching demand from one band to another. For example, as noted above, in round 16 EE chose to switch part of its demand from the 800 MHz band to 2.6 GHz paired. This revealed a preference at the relative prices in round 16, as set out in Figure B3.1.

EE's round 15 bid was for a larger package with 9,600 eligibility points, and its round 16 package was the smaller, constraining package at 5,400 eligibility points. EE's revealed preference by bidding for a smaller package in round 16 imposed a constraint on EE's bids in the supplementary bids round through the relative cap activity rule in accordance with the formula shown above. Specifically for this example, EE's supplementary bid amount for the larger package could not exceed its bid for the smaller package by more than the relative cap amount of £569 million shown in the bottom row. This is derived in the penultimate column as the difference in package prices when both were evaluated at clock prices in round 16 (and, for comparison, the column before that shows the bid amounts when EE bid for the larger package in round 15 in the first row, and for the smaller package in round 16 in the second row). In fact, the difference between the supplementary bids in the final column which EE chose to make was comfortably less than this constraint as it was only £388 million, between £1,652 million for the larger package and £1,264 million for the smaller package.[19]

The relative cap activity rule does not necessarily provide incentives for an operator to bid on its most preferred package in each clock round, due to the implications for flexibility in supplementary bids. The regulator can impose tighter activity rules which may improve price and package discovery in the clock stage. For example, the 'final price cap' prevents an operator from making a higher supplementary bid for its final clock package than in the clock stage, and links the maximum bid amount in all supplementary bids to the operator's final clock round bid.[20] Or a full-blown revealed preference rule would require consistency with all bids made by a bidder, not just the clock rounds in which eligibility was dropped – this approach was used in Canada's 2019 auction for the 600 MHz band.[21] The absence of tighter activity rules in the UK's 2013 auction contributed to its limited price and package discovery in the clock stage, and to the outcome being entirely dependent on the supplementary bids round. However, tighter activity rules can have disadvantages as well as strengths (see Section 8.4).

Setting eligibility points

The relative eligibility points set by the regulator for lot categories affect how an operator can switch its bids between them during the clock rounds. One reason for a bidder to switch between categories as relative prices change is if it considers them to be substitute spectrum. In the UK's 2013 auction, the categories of 800 MHz with and without coverage obligation (A2 and A1 respectively) were spectrum in the same band, so they were very close substitutes. Movements in the relative prices of categories A1 and A2 provided different discounts for taking on the coverage obligation. In addition, the categories of 2.6 GHz paired and unpaired (C and E) provided alternative capacity spectrum, although before the auction it was less clear how close substitutes these two categories would be, because the technology for unpaired spectrum was little used in Europe at that time.

A second reason for a bidder to switch between categories is based on its budget. For instance, an operator may have demand for coverage spectrum (800 MHz) and capacity spectrum (2.6 GHz), but a limited budget and so be unable to acquire both. It may prioritise winning the coverage spectrum, for example, but if that becomes too expensive relative to its budget, it could switch its demand to the cheaper capacity spectrum.

Eligibility point ratios between categories are sometimes set by regulators on the basis of expected relative value, such as using the ratio of reserve prices. This was the starting point for Ofcom's choices for the 2013 auction. However, the ratio of reserve prices is often unsuitable when applied to substitute inputs as illustrated by categories A1 and A2 for the 800 MHz band.[22] Comparing the same amount of spectrum, the reserve prices for 20 MHz were £450 million for A1 and £250 million for A2, a ratio of 1.8 (the difference in reserve prices reflecting a crude estimate by the regulator of the cost of the coverage obligation attached to the A2 lot). If the eligibility points had been set using this ratio, an operator who was only bidding for 800 MHz spectrum and switched from A1 to A2 during the clock stage would have faced a reduction in its relative eligibility from 1.8 points to 1 point. It would therefore have been unable to switch back to A1 if the relative price of A2 went up, except by bidding for less spectrum, 10 MHz instead of 20 MHz with eligibility of 0.9 points. It would then have not had enough eligibility to switch again into A2, if the relative price of A1 increased.

In essence, it can be 'putting the cart before the horse' to use reserve prices to fix the eligibility points and the terms of switching between categories of closely substitutable spectrum. It is the role of the auction to set prices reflecting market value, and there can be a high risk of regulatory failure for the regulator to second-guess this process. Again, the 800 MHz band provides an illustration,

Figure B3.2. Eligibility points per 10 MHz in 2013, based on relative reserve prices except for 800 MHz spectrum with and without the coverage obligation

Category	Label	Reserve price per 10 MHz	Eligibility points per 10 MHz
800 MHz (with coverage obligation)	A2	£100m	2250
800 MHz (without coverage obligation)	A1	£225m	2250
2.6 GHz paired	C	£15m	150
2.6 GHz unpaired	E	£0.2m	2

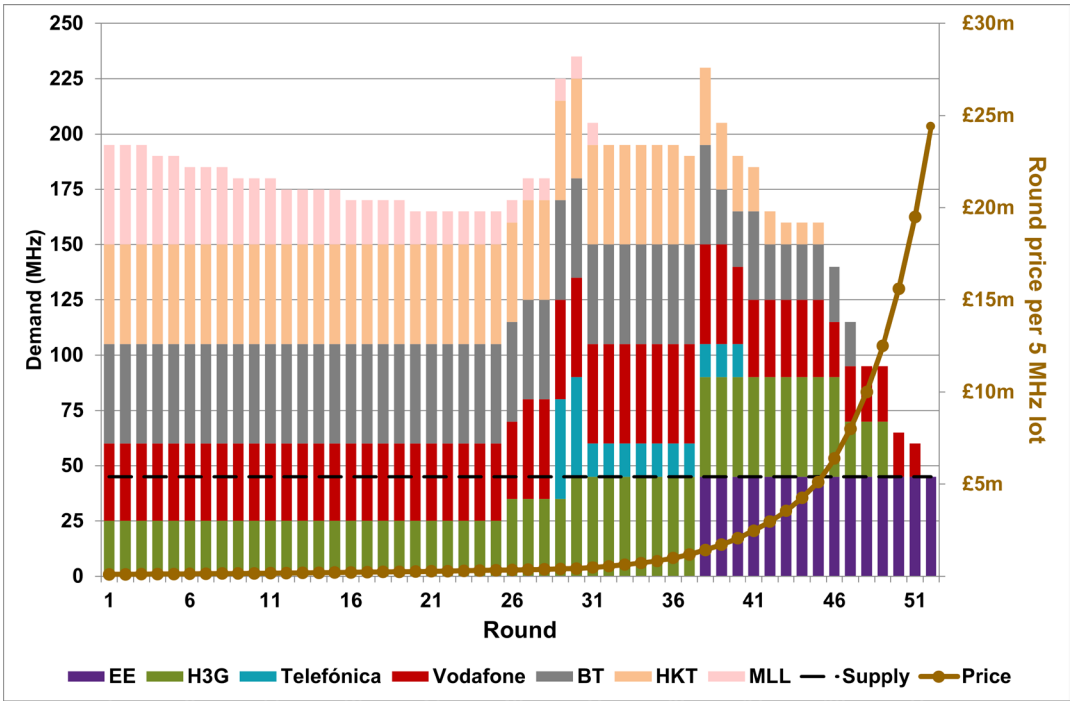
Source: Author from Tables 7.1 and 8.1 in Ofcom (2012b).

because Ofcom decided to depart from pre-auction estimates of relative value when setting these eligibility points and specified the same eligibility per MHz for A1 and A2 lots. The principle adopted here was to reflect suitable *relative amounts* of spectrum for substitute spectrum, in this case MHz for MHz between 800 MHz spectrum with and without the coverage obligation. This approach facilitated a significant amount of switching in both directions between A1 and A2 during the clock rounds, as shown in Figure B1.2 in the movement between the paler and darker colour shades for each bidder (with and without the coverage obligation). At the end of the clock stage, such switching led to the exact same price per MHz, which was extremely different from the regulator's pre-auction estimate for reserve prices.

The ability of bidders to switch in *both* directions can effectively be precluded by eligibility point choices as was the case for other bands in 2013. Ofcom set the ratios of eligibility points between the categories for 800 MHz and 2.6 GHz paired, and those for 2.6 GHz paired and unpaired, by using relative reserve prices. Figure B3.2 compares reserve prices and eligibility points for the four lot categories (all expressed per 10 MHz for ease of comparison). In the first two rows 800 MHz with and without the coverage obligation had the same eligibility points, even though their reserve prices were quite different. Comparing the second and third rows shows that the ratio of eligibility points between 800 MHz without coverage obligation and 2.6 GHz paired was the same as their ratio of reserve prices. This was also the case for 2.6 GHz paired and unpaired in the third and fourth rows.

Consequently, operators could switch from 800 MHz to 2.6 GHz paired, or from paired to unpaired 2.6 GHz, but it would involve such a large reduction in eligibility that switching back was not feasible.[23] So the regulator's choices for eligibility points contributed to one-way switching during the clock stage, and the lowest-value band, 2.6 GHz unpaired, was the last to be resolved. For example, as shown in Figure B3.3, the highest levels of demand for 2.6 GHz unpaired were in clock rounds 30 and 38, due to Telefónica, H3G, and EE switching demand into this band from other categories. However, subsequently there was no switching away by an operator from 2.6 GHz unpaired into other categories, and only demand reductions. After the clock stage, operators could still use their package bids in the supplementary bids round to express their preferences for substitute spectrum. But the limitations on switching to and from the 800 MHz, 2.6 GHz paired, and 2.6 GHz unpaired bands during the clock stage likely contributed to weak price discovery in the 2013 auction. Operators could have had greater opportunities to switch their demand between bands in 2013, if the ratios of eligibility points set by the regulator had departed from the ratios of reserve prices and instead attempted to reflect suitable relative amounts of spectrum.

Figure B3.3. Clock bids and prices in the 2.6 GHz unpaired band in 2013 (illustrating one-way switching)



Source: Author from Ofcom auction documents.

Notes

[1] In addition, category D was for concurrent low-power licences for a portion of the 2.6 GHz paired spectrum (and category B allowed for 1800 MHz spectrum that was ultimately not included in the auction). The D1 and D2 lot categories included up to 10 concurrent low-power licences for 20 MHz and 40 MHz respectively of the 2.6 GHz paired spectrum. The auction bidding determined whether this spectrum was awarded as these shared low-power licences or instead as exclusive high-power licences in category C (which was the outcome).

[2] Lower principal stage prices compared to the clock rounds reflected several factors, including subtleties in the determination of second prices in package auctions. There were 'missing bids' in the highest losing combinations to determine second prices, resulting in reserve price components in all package prices apart from BT's. In addition, there had been excess supply at the end of the clock stage in the 800 MHz and 2.6 GHz paired bands, and principal stage prices were non-linear with declining marginal prices (except over amounts reflecting synergies in losing bids).

[3] Linear reference prices are the linear prices (that is, the same per MHz) that are estimated to be closest to market-clearing prices. In the case of the 2013 auction, linear prices would have failed to clear the market due to synergies (non-linear bid values), resulting in either excess demand or excess supply in each band according to the bids made. For further details, and other methods which could be used to decompose package prices into band-specific prices, see Ofcom (2015, annex 6).

[4] Vickrey (1961).

[5] Ausubel and Milgrom (2006), and Rothkopf (2007).

[6] Marszalec (2018) shows experimental evidence consistent with collusion in Vickrey auctions.

[7] In assignment stages and the CCA principal stage, the core is with respect to the preferences reported in the auction bids. It may be different from the core reflecting true values if bids are not straightforward.

[8] Day and Milgrom (2008).

[9] Erdil and Klemperer (2010).

[10] Day and Cramton (2012).

[11] In theory, a winning bidder can reduce its price by lowering its winning bid in order to change either the core or other winners' Vickrey prices and so shift the set of prices in the minimum-revenue core that is nearest to Vickrey — see Erdil and Klemperer (2010). However, in practice the number of categories and bidders usually makes it very difficult to identify the precise bidding strategy to achieve the price reduction. Furthermore, the strategy is usually not risk-free, because it could result in the bidder winning a less preferred package, or failing to win. The risk-reward trade-off may be unfavourable and so gaming of the reference rule, although possible, is often not attractive in practice. For example, limited bid shading was observed in the experiments by Bichler, Shabalin and Wolf (2013).

[12] Ausubel and Baranov (2020a, p.252).

[13] Ofcom (2015, table A6.5).

[14] An exception may be some instances of successful coordination in the assignment stage of the forward auction in the USA's 2016–17 incentive auction, which used Vickrey pricing (not minimum-revenue core) – see Kominers and Teytelboym (2020, p.1186).

[15] For example, the relaxed activity rule during the clock stage, and the relative and final price cap rules in the supplementary bids round were all included in the design for Ireland's auction planned for 2021 (but delayed by litigation) – see ComReg (2021, paragraphs 3.205–3.209 and 3.221–3.226).

[16] Bichler and Goeree (2017, section 3.2.1).

[17] The underlying logic is that, if an operator fails to bid straightforwardly in the clock stage, it might be prevented by the activity rules from placing its preferred bids in the supplementary bids round.

[18] See Regulation 43(7) in the 2013 auction rules - The Wireless Telegraphy (Licence Award) Regulations, Statutory Instruments, 2012 No. 2817, Electronic Communications: https://perma.cc/NDS9-N7NB.

[19] Figure B3.1 provides a simplified explanation of the relative cap rule for larger packages, because if, as here, the constraining package was not the final round package, then a chain of bid restrictions applied. The supplementary bid on a larger package was capped relative to the supplementary bid on the constraining package, which in turn was capped relative to another constraining package. The chain ended when the constraining package was the final clock package.

[20] Bichler and Goeree (2017, section 3.2.2).

[21] Ausubel and Baranov (2020b).

[22] Even for budget-based switching, the suitability of eligibility points based on relative reserve prices depends on bidders having demand for a similar number of lots in each category, and also the regulator setting reserve prices in a similar fashion between the relevant bands (which was not the case in the UK's 2013 auction, as explained in Section 7.3).

[23] Ausubel and Baranov (2020b, section 7.3).

References

Note: means an open access publication.

Ausubel, Lawrence and Baranov, Oleg (2020a) 'Core-Selecting Auctions with Incomplete Information', *International Journal of Game Theory*, 49(1):251–273: https://doi.org/10.1007/s00182-019-00691-3

Ausubel, Lawrence and Baranov, Oleg (2020b) 'Revealed Preference and Activity Rules in Dynamic Auctions', *International Economic Review*, 61(2): 471-502: https://doi.org/10.1111/iere.12431

Ausubel, Lawrence and Milgrom, Paul (2006) 'The Lovely but Lonely Vickrey Auction', in *Combinatorial Auctions*, Cramton, P, Shoham, Y and Steinberg, R (eds), MIT Press, Chapter 1

Bichler, Martin and Goeree, Jacob (2017) 'Frontiers in spectrum auction design', *International Journal of Industrial Organization*, 50: 372–391: https://doi.org/10.1016/j.ijindorg.2016.05.006

Bichler, Martin; Shabalin, Pasha; and Wolf, Jürgen (2013) 'Do core-selecting Combinatorial Clock Auctions always lead to high efficiency? An experimental analysis of spectrum auction designs', *Experimental Economics*, 16(4): 511-545: https://doi.org/10.1007/s10683-013-9350-3

ComReg (2021) 'Multi Band Spectrum Award – Information Memorandum and Draft Regulations, The 700 MHz Duplex, 2.1 GHz, 2.3 GHz and 2.6 GHz Bands', ComReg 21/40: https://perma.cc/55FC-ED3R

Day, Robert and Cramton, Peter (2012) 'Quadratic Core-Selecting Payment Rules for Combinatorial Auctions', *Operations Research*, 60(3): 588-603: https://doi.org/10.1287/opre.1110.1024

Day, Robert and Milgrom, Paul (2008) 'Core-Selecting Package Auctions', *International Journal of Game Theory*, 36(3–4): 393–407: https://doi.org/10.1007/s00182-007-0100-7

Erdil, Aytek and Klemperer, Paul (2010) 'A New Payment Rule for Core-selecting Package Auctions', *Journal of the European Economic Association*, 8(2/3): 537–547: https://doi.org/10.1111/j.1542-4774.2010.tb00524.x

Kominers, Scott Duke and Teytelboym, Alexander (2020) 'The Parable of the Auctioneer: Complexity in Paul R. Milgrom's *Discovering Prices*, *Journal of Economic Literature*, 58(4): 1180–1196: https://doi.org/10.1257/jel.20191504

Marszalec, Daniel (2018) 'Fear Not the Simplicity - An Experimental Analysis of Auctions for Complements', *Journal of Economic Behavior and Organization*, 152: 87–97: https://doi.org/10.1016/j.jebo.2018.04.023

Ofcom (2012b) 'Assessment of future mobile competition and award of 800 MHz and 2.6 GHz', Statement, 24 July. https://perma.cc/36NV-68FF

Ofcom (2015) 'Annual licence fees for 900 MHz and 1800 MHz spectrum', Statement, 24 September. https://perma.cc/BW4F-PTCC

Ofcom auction documents, 'Spectrum awards archive'. https://www.ofcom.org.uk/spectrum/spectrum-management/spectrum-awards

Rothkopf, Michael (2007) 'Thirteen Reasons Why the Vickrey-Clarke-Groves Process Is Not Practical', *Operations Research*, 55(2): 191–197: https://doi.org/10.1287/opre.1070.0384

Vickrey, William (1961) 'Counterspeculation, Auctions, and Competitive Sealed Tenders', *Journal of Finance*, 16(1): 8–37: https://doi.org/10.2307/2977633